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FINITE ELEMENT ANALYSIS OF REINFORCED EMBANKMENT FOUNDATION

A. VARADARAJAN*, K. G. SHARMA AND M. A. A. ALY

Department of Civil Engineering, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi 110 016, India

SUMMARY

A finite element analysis of a reinforced embankment-foundation system has been conducted using a coupled formulation and elastoplasticity theory. Such important factors as type of reinforcement, the type of clay, depth of foundation and drainage condition affecting the system have been systematically investigated using appropriate constitutive models to depict various components of the system and material parameters of two typical soft clay deposits found in India. The displacements, reinforcement force and maximum heights of the embankments are among the aspects presented and discussed. It is shown that the effectiveness of the reinforcement is dependent on its stiffness and the shear strength of the clay deposit. The foundation depth has significant effect on the nature and magnitude of displacement, the reinforcement force and the height of embankment. Drainage conditions are shown to markedly influence the effectiveness of reinforcement. Copyright © 1999 John Wiley & Sons, Ltd.

Key words: finite element; elastoplastic; coupled analysis; reinforced-embankment foundation

1. INTRODUCTION

Soft clay deposits do not often have adequate shear strength to support embankments. Many of the ground improvement techniques, which have been used in the past to increase the shear strength of the soil, are time-consuming and uneconomical. The emergence of geosynthetic reinforcement in recent times has revolutionized the concept of ground improvement.

A geosynthetic reinforcement in sheet or grid form is first placed on the foundation soil before an embankment is constructed over it. The development of shear stress at the soil–reinforcement interface and the tension in the reinforcement provide resistance to failure, thus improving the stability of the embankment-foundation system. The mechanism of load-sharing and/or transfer among different elements, viz., embankment, soil-reinforcement interfaces, reinforcement and foundation is complex and is influenced by the properties of the individual elements as well as the relative magnitudes of the properties with respect to each other. Analyses based on failure modes, viz., bearing capacity failure, sliding failure, squeezing failure and rotational failure, are simplified and do not provide an integrated picture of stress–strain/deformation behaviour of the complete system. An alternative approach is to use the finite element method, FEM, which not only can model the behaviour of various components of the reinforced embankment system and loading

*Correspondence to: A. Varadarajan, Department of Civil Engineering, Indian Institute of Technology, Hauz Khas, New Delhi 110 016, India

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conditions but also can provide complete details of the stress–strain/deformation behaviour of the entire system. A brief review of the studies conducted using FEM is presented herein.

Rowe[1] and Rowe and Soderman[2–4] used a Mohr–Coulomb yield criterion to analyse geotextile reinforced embankments up to failure with undrained behaviour of the foundation.

Hird and Kwok[5] and Hird *et al.*[6] studied the effect of embankment fill strength and foundation depth on the performance of reinforced embankments with narrow crest width. The embankment soil was modelled with Mohr–Coulomb yield criterion. Undrained behaviour of the underlying foundation soil was modelled with Modified Cam-clay model. The reinforcement was characterized as a linear or bilinear elastic material.

Jonathan *et al.*[7] investigated the effectiveness of using geosynthetic tensile reinforcements for strengthening two test embankments. The behaviour of the embankment fills and the foundation materials was simulated using the Duncan–Chang hyperbolic model. The reinforcement was treated as a linear elastic material.

Tavassoli and Bakeer[8] analysed the performance of an embankment (under working loads) with the geotextile as reinforcement at its base using a Modified Cam-clay model. The stiffness modulus of the embankment fill material was assumed to be constant.

These studies have aimed at investigating a few factors affecting the reinforced embankment-foundation system. The behaviour of various elements is characterized, in certain studies, with simplified models such as elastic behaviour for reinforcement and the Mohr–Coulomb criterion for the foundation clay.

2. SCOPE

This paper deals with the finite element analysis of a reinforced embankment-foundation system. Various components of the system are characterised by appropriate constitutive models using elasto-viscoplasticity. The elasto-viscoplasticity algorithm proposed by Zienkiewicz and Cormeau[9] has been adopted. Elasto-viscoplasticity has been used as an artifice to obtain elasto-plastic solutions. A computer program has been developed using a coupled formulation to facilitate analysis with undrained, drained and partially drained conditions. Included in it is the simulation of construction of embankment. A comprehensive study has been conducted to investigate the effects of the type of reinforcement, the type of clay, the depth of foundation and the drainage conditions, viz., undrained, drained and partially drained conditions.

3. REINFORCED EMBANKMENT-FOUNDATION

A typical highway embankment of 18 m crest width and 1 : 2 side slope has been chosen for the study (Figure 1). The embankment of height H rests on a soft clay foundation of depth D underlain by a rigid rock deposit. A single layer of geotextile reinforcement is used at the interface between the embankment and the clay foundation. The water table is located at the ground surface.

4. DISCRETIZATION

The discretization of the reinforced embankment-foundation along with the boundary conditions are shown in Figure 2. Three types of elements, viz., (i) eight-noded quadrilateral solid elements

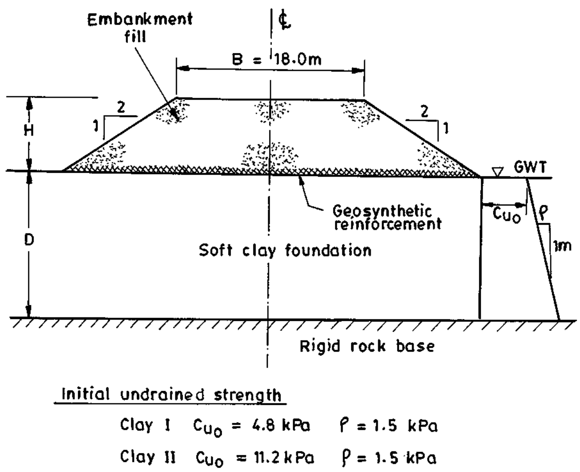

Figure 1. Reinforced embankment–foundation system

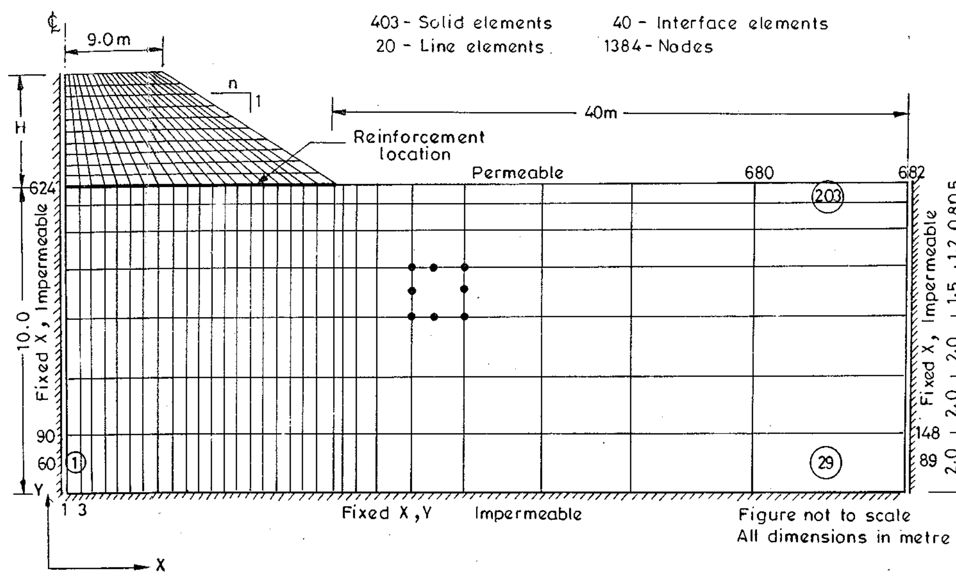

Figure 2. Finite element discretisation of the embankment–foundation system

for the soil, (ii) zero thickness six-noded joint elements for the soil-reinforcement interfaces and (iii) three-noded bar elements for the reinforcement have been used.

5. MATERIAL MODELS AND PARAMETERS

From the literature survey on soft clays in India,[10] two clay deposits, Kerala clay (Clay I) and Madras clay (Clay II), found in coastal parts of South India, representing weak and strong clays,

respectively, have been chosen. Clay I has natural water content, 99–145 per cent, Liquid Limit, 104 –150 per cent, Plastic Limit, 31– 48 per cent and organic content, 9·5–11·6 per cent. Clay II has natural water content, 60 –75 per cent, Liquid Limit, 65–86 per cent, Plastic Limit, 24 –28 per cent and organic content, 0·2– 0·8 per cent. Clay I has been adopted in all the studies, whereas Clay II has been used to study the effect of type of foundation clay only. The soft clay foundation has been modelled as an elasto-plastic Modified Cam-clay model.[11] The behaviour of the embankment fill consisting of sandy soil has been represented by an elastic–perfectly plastic material with Mohr–Coulomb yield criterion and fully non-associative flow rule; the Young's modulus for the fill is dependent on stress as proposed by Janbu.[12] The geosynthetic reinforcement has been characterized with elasto-plastic behaviour using von-Mises yield criterion. The geosynthetic reinforcement adopted represents commonly used materials such as Advance Type I geotextile, stabilenka 200, high tenacity woven polyester geotextile, stabilenka 400 and Tensar SS1, SR1 and SR2. The properties for the reinforcement have been chosen based on the data available in literature (for example, References 1–7).

For interface behaviour between the soil and reinforcement, the Mohr–Coulomb criterion has been adopted. The strength at the lower interface is governed by the shear strength of the foundation soil surface, whereas the upper surface behaviour is controlled by the shear strength of the fill material. The material parameters used for various components are given in Tables I and II.

Table I. Properties and strength parameters used for embankment fill material and clay foundation

Material	Properties used in the analysis	
Embankment fill		
	Unit weight, $\gamma_f = 20\,kN/m^3$	
	Cohesion, $c_f = 0$	
	Angle of internal friction, $\phi_f = 40^\circ$	
	Janbu's parameter, $K = 150$	
	Janbu's parameter, $m = 0{\cdot}5$	
	Poissons ratio, $\nu_f = 0{\cdot}35$	
Clay foundation	Kerala Clay (Clay I)	Madras Clay (Clay II)
Submerged unit weight,	$\gamma' = 4$	$7\,kN/m^3$
Angle of internal friction,	$\phi' = 29^\circ$	35°
Initial void ratio,	$e_0 = 3{\cdot}92$	1·9
Compressibility index,	$\lambda = 0{\cdot}83$	0·27
Swelling index,	$\kappa = 0{\cdot}13$	0·05
Young's modulus,	$E' = 3500$	8000 kPa
Poissons ratio,	$\nu' = 0{\cdot}3$	0·3
In situ Stress ratio,	$K_0 = 0.52$	0·58
Preconsolidation pressure,	$P_{co} = 40$	90 kPa
	For undrained analysis	
Apparent bulk modulus	$K_a = 3{\cdot}5 \times 10^5$	8×10^5 kPa
	For coupled consolidation analysis	
Permeability in vertical direction,	$k_v = 2 \times 10^{-5}$ m/day	—
Permeability in horizontal direction,	$k_h = 3 \times 10^5$ m/day	—

Table II. Properties and strength parameters used for interface elements and reinforcement

Material	Properties used in the analysis	
Interface elements	*Fill-reinforcement interface*	
Adhesion	$C_a = 0$	
Interface friction angle	$\delta_f = \phi_f = 40^\circ$	
Shear stiffness	$K_s = 2000\,\mathrm{kN/m^3}$	
Normal stiffness	$K_n = 3 \times 10^6\,\mathrm{kN/m^3}$	
	Clay-reinforcement interface	
	(Clay I)	(Clay II)
Adhesion	$C_a = C_{uo} = 4{\cdot}8$	$11{\cdot}2\,\mathrm{kPa}$
Interface friction angle	$\delta_f = 0$	0
Shear-stiffness	$K_s = 2000$	$2000\,\mathrm{kN/m^3}$
Normal stiffness	$K_n = 3 \times 10^6$	$3 \times 10^6\,\mathrm{kN/m^3}$
Reinforcement	Stiffness J(kN/m)	Tensile strength σ_0(kN/m)
	200	8
	1000	80
	2000	140
	4000	260
	8000	480

6. DETAILS OF THE ANALYSIS

Four series of analyses have been conducted to study the effect of (a) the type of reinforcement, (b) and the type of clay, (c) the depth of foundation and (d) the drainage condition.

Three foundation depths, viz., $D = 2{\cdot}5$, 6 and 10 m have been identified in the study keeping in view the range of depths which is generally of importance. In all other cases, a foundation depth of 10 m has been adopted.

All the above studies have been conducted under undrained conditions of foundation. The effect of drainage condition has been studied with three drainage conditions, viz., undrained, drained and partially drained conditions.

For each embankment–foundation configuration, an embankment height, H_0 has been computed without reinforcement. This height corresponds to the failure condition as defined in the following paragraph. This height has been identified in the present study as a common datum to compare the effect of various reinforcements on the displacements and the stresses.

The embankment construction is simulated through ten lifts. In each lift the loading is applied in several increments. As many as 250 load increments have been used in each case. A number of iterations have been carried out for each load increment to ensure convergence. The total number of iterations required have often added up to 1500. The failure of the embankment has been defined as the height at which the increment in vertical displacement is equal to or more than the present increment in fill thickness.[4]

The coupling of the stress–deformation analysis with consolidation has been formulated using Biot's theory[13,14] to study the effect of the partial drainage condition. A computer program has

been developed incorporating all the above features and validated with standard results available in literature.

7. RESULTS AND DISCUSSIONS

7.1. Effect of type of reinforcement

The variation of horizontal displacement and vertical displacement on the surface of the foundation is presented in Figures 3 and 4 at H_0 ($= 3$ m). It is clearly seen that the deformation of

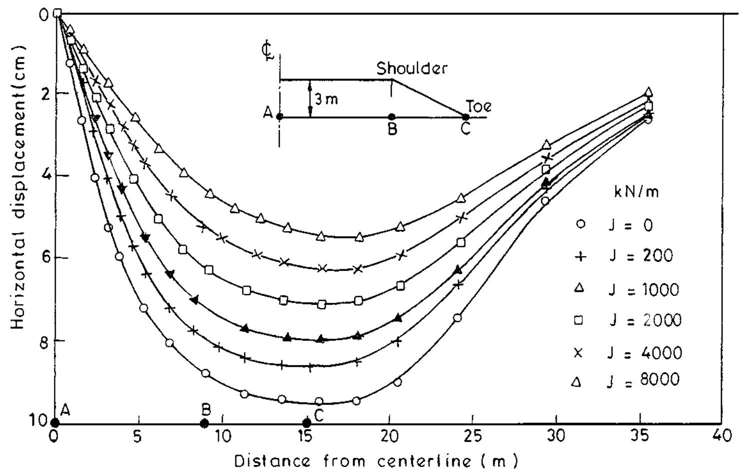

Figure 3. Variation of horizontal displacement of the foundation surface for various reinforcements at $H_0 = 3$ m

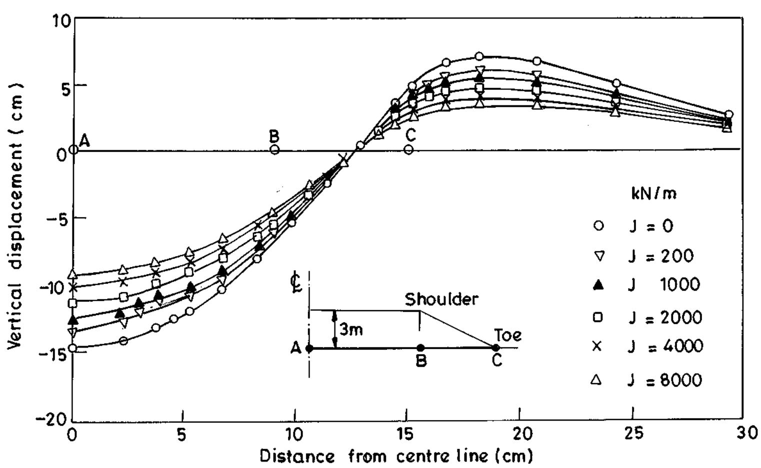

Figure 4. Variation of vertical displacement on the foundation surface for various reinforcements at $H_0 = 3$ m

foundation surface decreases with each increase in reinforcement stiffness. The maximum effect of reinforcement on the horizontal displacement is noted near the embankment toe. The effect of reinforcement on vertical displacements is found at the centre line and away from the toe. Figure 5 shows the variation of maximum horizontal and vertical displacements with reinforcement modulus value for Clay I. It is noted that the effect of reinforcement is significant up to $J = 4000\,\mathrm{kN/m}$ where the percentage reductions of the horizontal and vertical deformations are 35 and 30 per cent, respectively. The increase of J to 8000 kN/m results in a further reduction of only 12 and 9 per cent in the displacements. Similar observations have been reported by Rowe and Soderman[3] for cases of reinforcement stiffness up to 4000 kN/m. In their study, Mohr–Coulomb yield criterion has been used to model the soil behaviour.

Figure 6 shows the variation of the tensile force in the reinforcement at $H_0 = 3\,\mathrm{m}$ for various reinforcement stiffness values. The tensile force increases from toe to the central line. It also increases with the reinforcement stiffness. The reinforcement force is almost proportional to the height of the embankment. Similar observations have been reported by Tavassoli and Bakeer.[8] It may, however, be noted that the nature of variation of reinforcement force will depend on such factors as depth of foundation, type of clay and embankment geometry. The tensile reinforcement forces developed in this case and subsequent cases reported in this paper are less than the tensile strengths of reinforcements.

7.2. *Effect of the type of clay*

The H_0 values for the Clay I and Clay II are 3 and 5 m, respectively, and are consistent with the strengths of the two clays. The nature of distribution of displacements and stresses in the two clay deposits are similar for various reinforcing materials. Figure 5 also shows the variation of

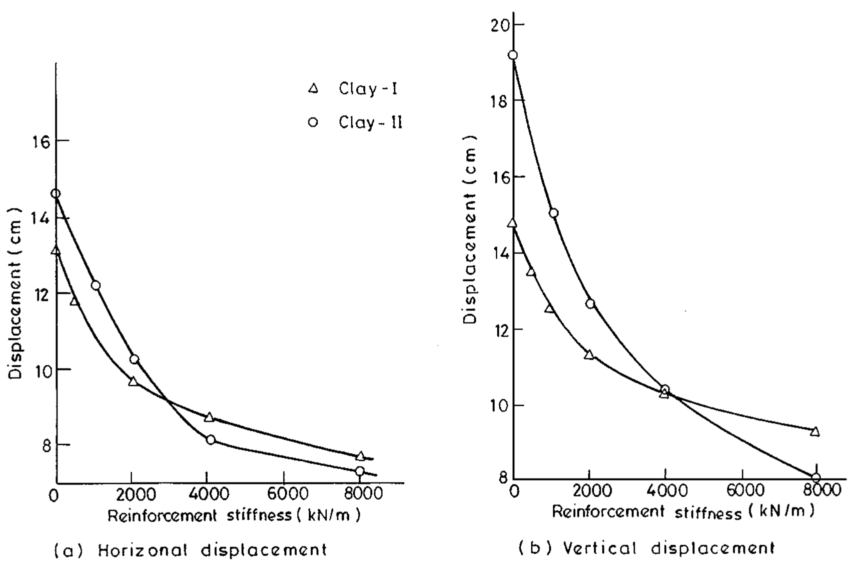

Figure 5. Variation of maximum displacement with increase in reinforcement stiffness at $H_0 = 3\,\mathrm{m}$ for case I and case II

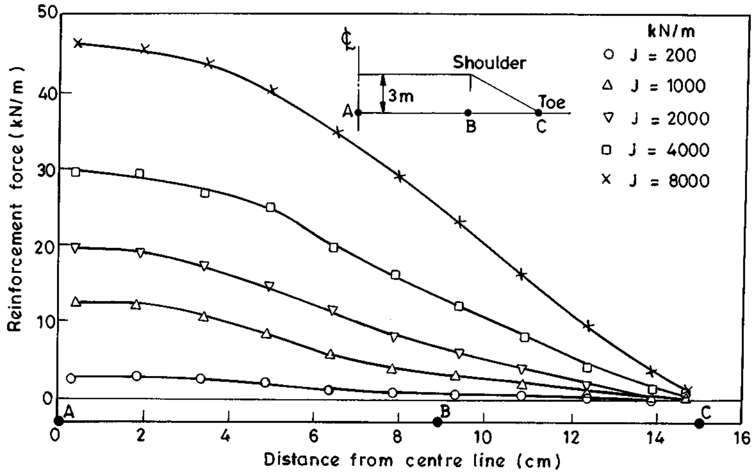

Figure 6. Variation of the tensile force in the reinforcement at $H_0 = 3$ m for various stiffness values

maximum horizontal and vertical displacements with reinforcement modulus for Clay II. It is noted that the effect of reinforcement in reducing the displacement is similar in the two deposits. The reduction, however, is more for the stronger clay deposit. From the comparison of the increase of maximum tensile force with reinforcement modulus for the two clays, it is observed that the stronger clay induces more force as found by Rowe and Soderman.[3]

The maximum height of the embankment for each reinforcement was computed for the two clay types. The maximum height of the embankment increases from 3 m to 3·5 m for Clay I and from 5 to 6·9 m for Clay II as the value of J increases from 0 to 8000 kN/m. The maximum height increases by about 12 and 20 per cent for Clay I and Clay II, respectively, as the value of j increases from 0 to 4000 kN/m. As the value of J increases from 4000 kN/m to 8000 kN/m the increase in height is about 4 per cent for Clay I and 15 per cent for Clay II. It is evident that the effectiveness of reinforcement to increase the maximum height of the embankment depends on the magnitude of shear strength of the clay and the strength at clay–reinforcement interface particularly for high reinforcement stiffness.

7.3. Effect of foundation depth

For the three foundation depths, viz., 2·5, 6·0 and 10 m, the maximum embankment heights H_0 without reinforcement are 3·8, 3·2 and 3 m, respectively.

The variation of horizontal and vertical displacements for various foundation depths are shown in Figures 7 and 8. It is observed that the foundation depth significantly affects the nature of variation of displacements. Whereas the maximum vertical displacement for a foundation depth of 2·5 m is near the shoulder, it is at the centre for the foundation depth of 10 m.

In Table III is shown the percentage reduction in maximum displacement due to reinforcement. It is seen that the effect of reinforcement is more for shallow foundation depths.

The distribution of reinforcement force for the three foundation depths is shown in Figure 9 for the reinforcement with $J = 4000$ kN/m. It is noted that the nature of distribution varies with

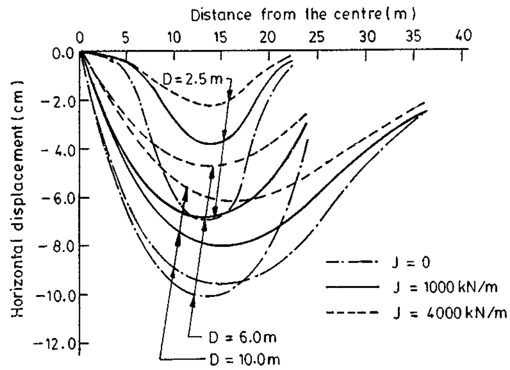

Figure 7. Variation of horizontal displacement of foundation surface for various foundations depths at H_0

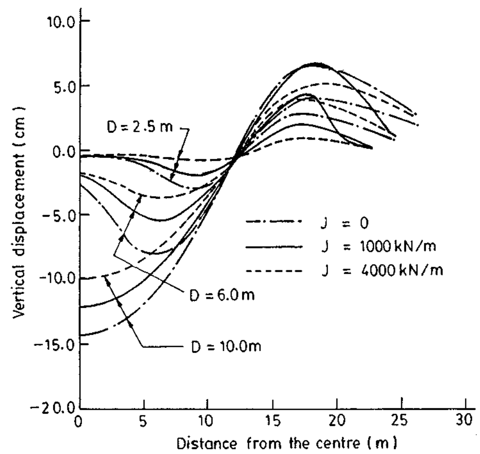

Figure 8. Variation of vertical displacement of foundation surface for various foundations depths at H_0

Table III. Percentage reductions of deformations in foundation soil for different foundation depths

Foundation depth (m)	Percentage reductions of deformations	
	Horizontal displacement	Vertical displacement
2·5	68	66
6·0	54	50
10·0	37	30

foundation depth. With increasing foundation depth, the reinforcement force increases for the range of embankment width/depth of foundation ratios examined.

In Figure 10 is shown the variation of maximum embankment height with foundation depth. It is observed that for smaller foundation depths (i) the maximum embankment height increases,

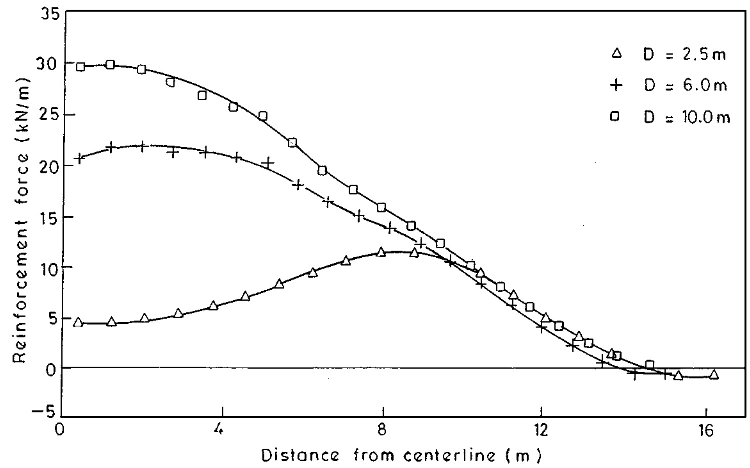

Figure 9. Variation of reinforcement force for various foundations depths with $J = 4000$ kN/m

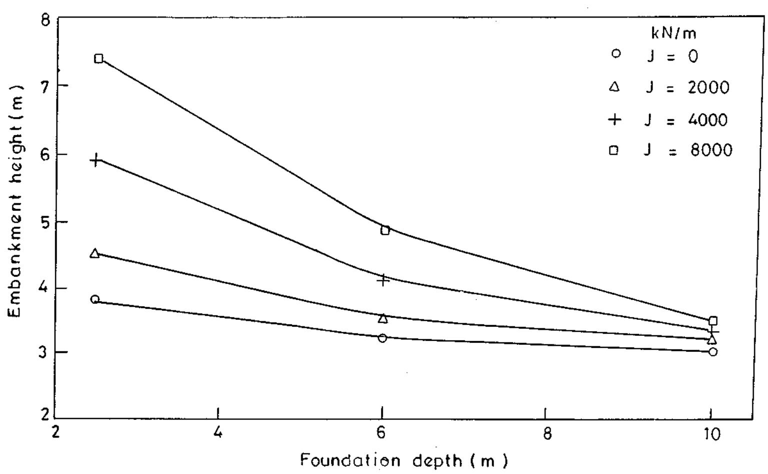

Figure 10. Variation of maximum embankment height with foundation depth

and (ii) the effectiveness of the reinforcement increases. Rowe and Soderman[4] have drawn similar conclusions using Mohr–Coulomb yield criterion to characterize the soil behaviour.

7.4. Effect of drainage condition

In the undrained analysis, no excess pore water pressure is allowed to dissipate and in the drained analysis, no excess pore water pressure is present. In the coupled analysis (partially drained case), a time duration of 24 days is adopted for the construction stage. After this stage, pore water pressure is allowed to dissipate with time. In all the three cases, the height of the embankment is 3 m ($= H_0$, the maximum embankment height under undrained conditions).

In Figures 11 and 12 are shown the variations of horizontal and vertical displacements on the foundation surface for the three drainage conditions. It is noted that the undrained condition gives maximum horizontal displacement and minimum vertical displacement. The reverse condition is seen for drained condition. The differences in displacements between unreinforced and reinforced cases decrease with drainage and they are negligible in the case of vertical displacement for fully drained condition. It is observed that, with drainage, the tensile force in the reinforcement decreases.

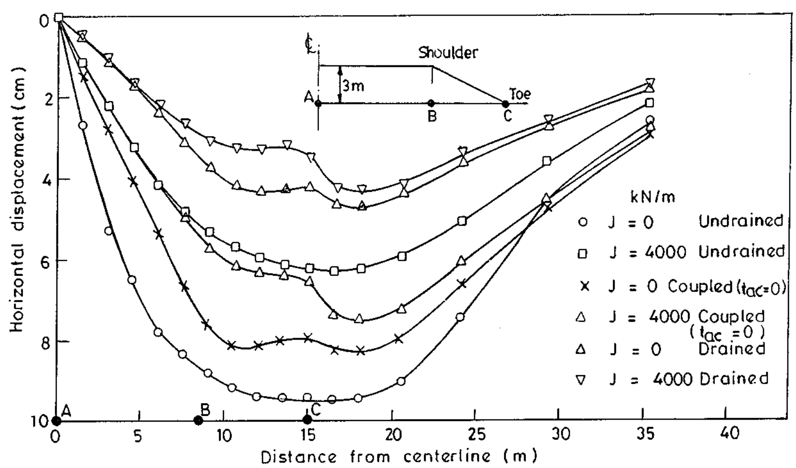

Figure 11. Variation of horizontal displacement on foundation surface for various drainage conditions

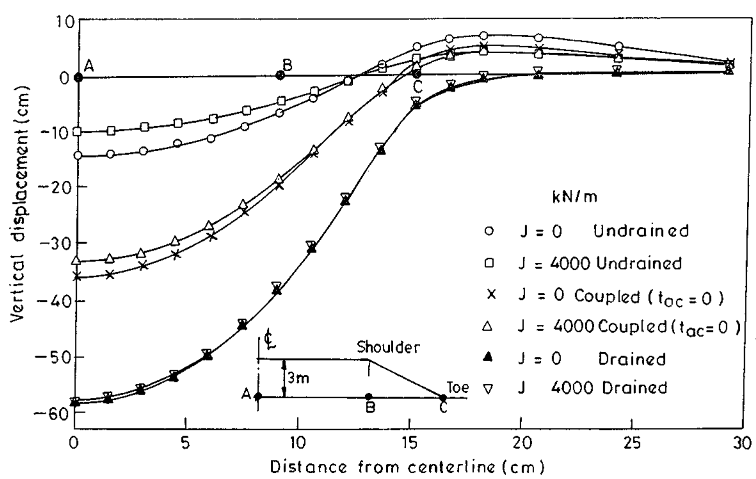

Figure 12. Variation of vertical displacement on foundation surface for various drainage conditions

8. CONCLUSIONS

A finite element analysis of a reinforced embankment–foundation system has been conducted using a coupled FEM formulation and elasto-plasticity theory. Four important factors affecting the system, viz., type of reinforcement, type of clay, depth of foundation and drainage condition have been studied using appropriate constitutive models and material parameters. The study provides a clear understanding of the behaviour of the system under various conditions. The following conclusions are drawn from the study.

The type of reinforcement as characterized by reinforcement stiffness, may cause significant reduction in displacements. The increase in the height of the embankment is, however, less marked with increase in the reinforcement stiffness. The effect of reinforcement is more pronounced at lower values of the reinforcement stiffness.

The type of clay as represented by the shear strength has significant effect in reducing displacements and increasing the maximum height of the reinforced embankment particularly at higher reinforcement stiffness values.

The foundation depth has considerable effect on both the distribution and the magnitude of displacements, the reinforcement force and the maximum height of the embankment. For smaller foundation depth, the effect of reinforcement stiffness is enhanced, as is the force in the reinforcement and the height of the embankment.

The effect of drainage results in the reduction of the effectiveness of the reinforcement in controlling the displacements.

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